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Monte-Carlo Simulations as a Tool to Understand Gamma-ray Coincidence Spectra: ^{22}Na Decay Inside a Well-counter

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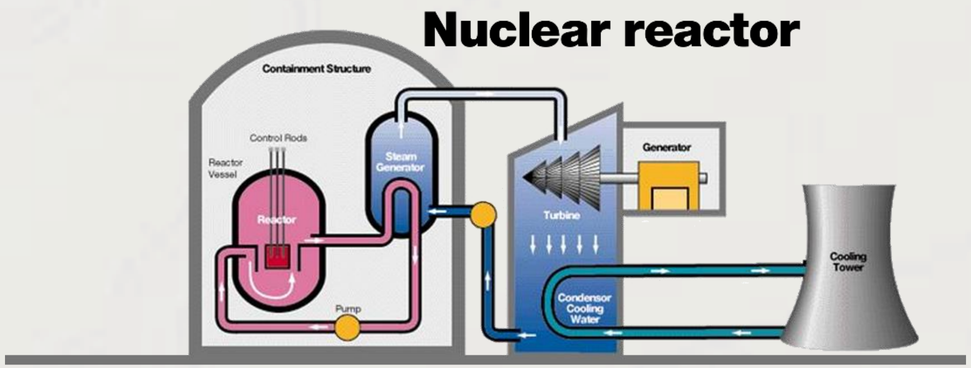
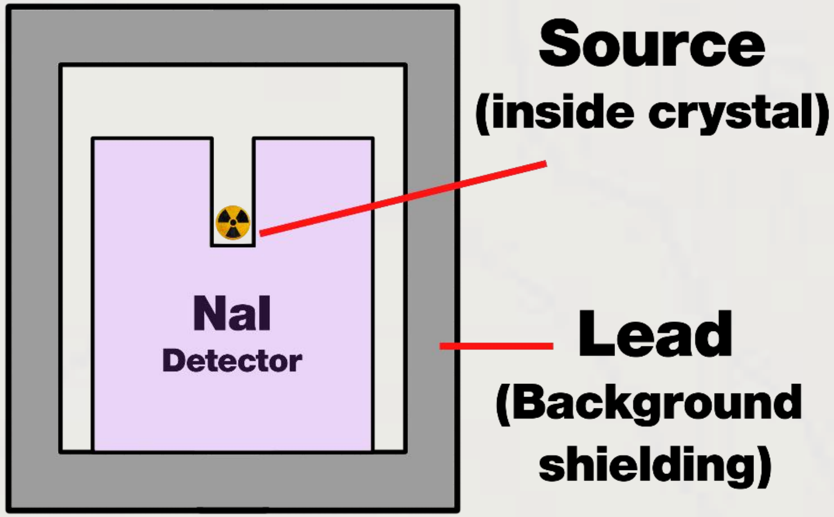
Origin of investigation

Koeberg experiments

- Long term measurements of a 50 kBq ^{22}Na source inserted in a well counter.
- Measure continuously for 732 hours
- Measure tiny differences in count rate





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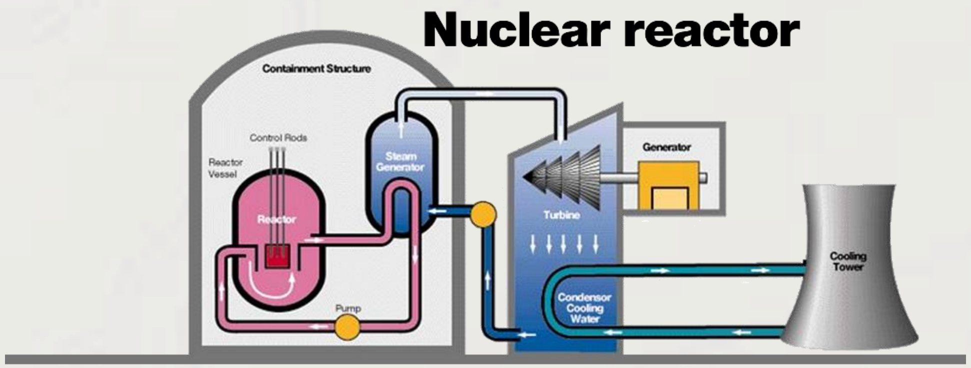
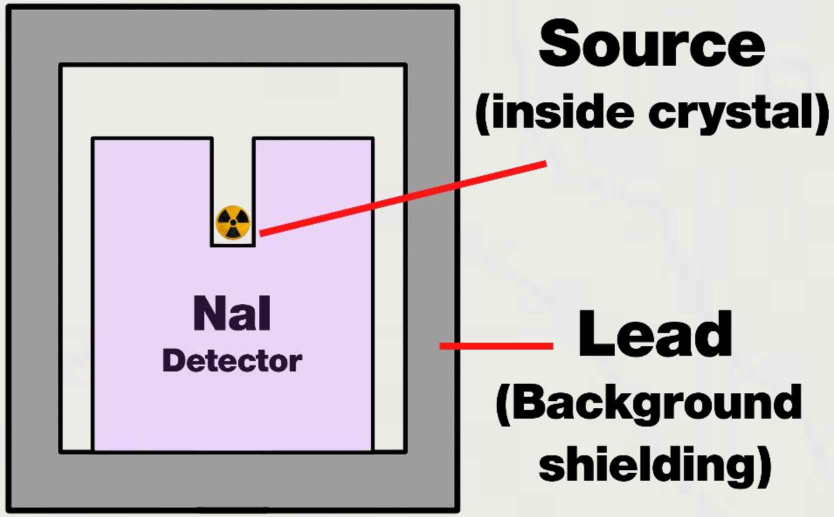
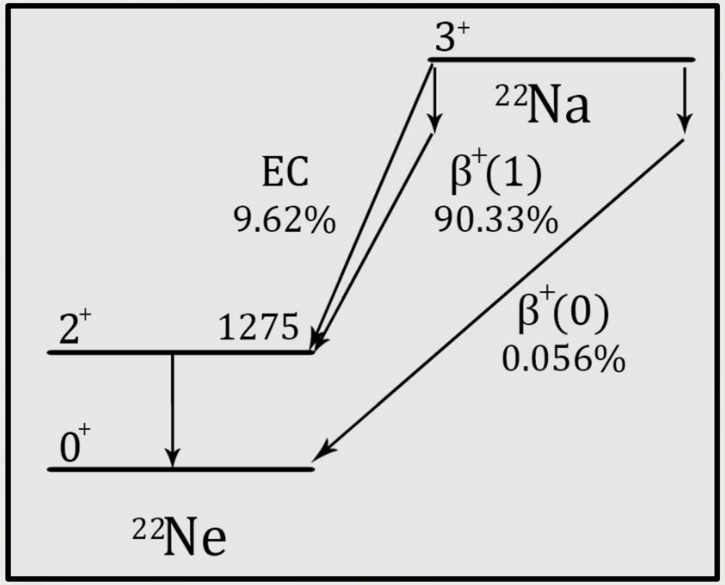
Goal

Understand and reconstruct measured spectra from Monte-Carlo simulations.





Decay scheme of ^{22}Na



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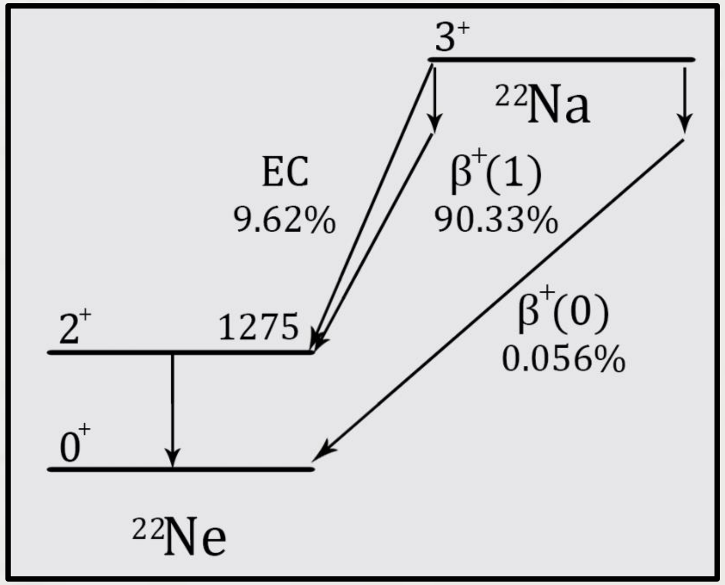

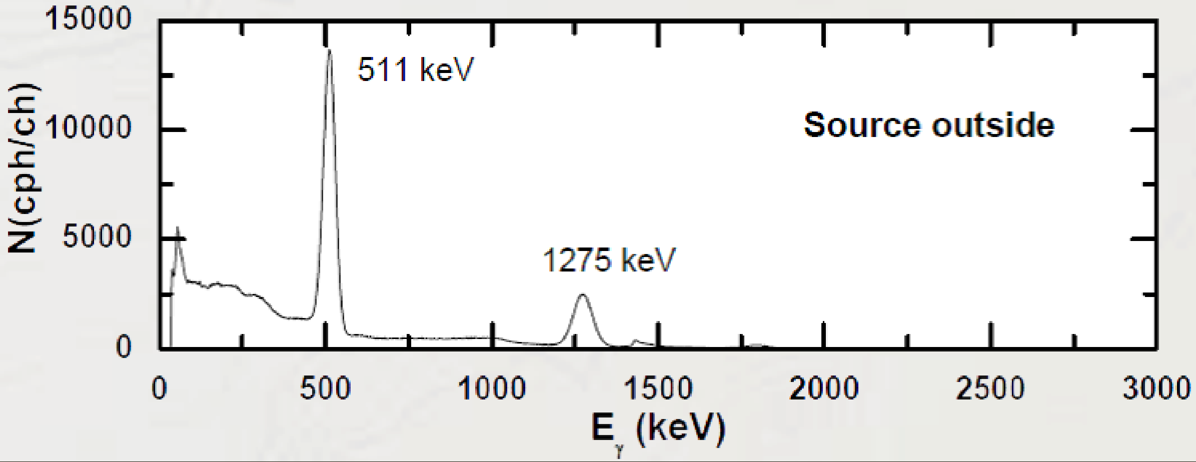

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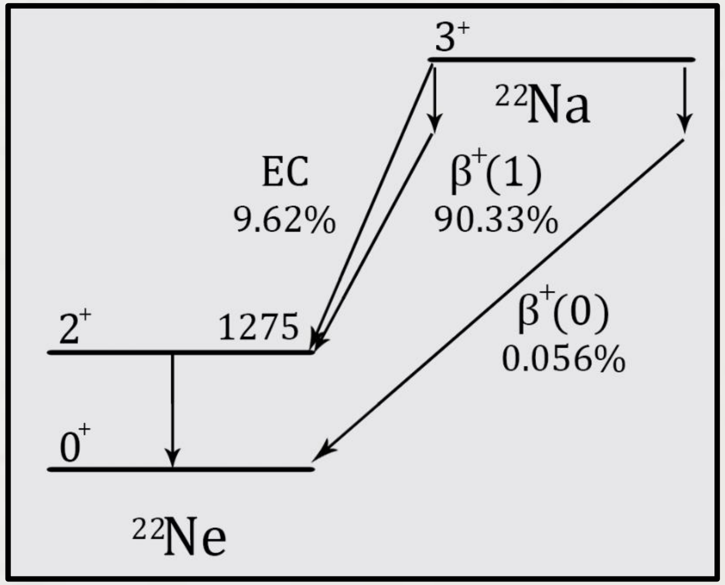

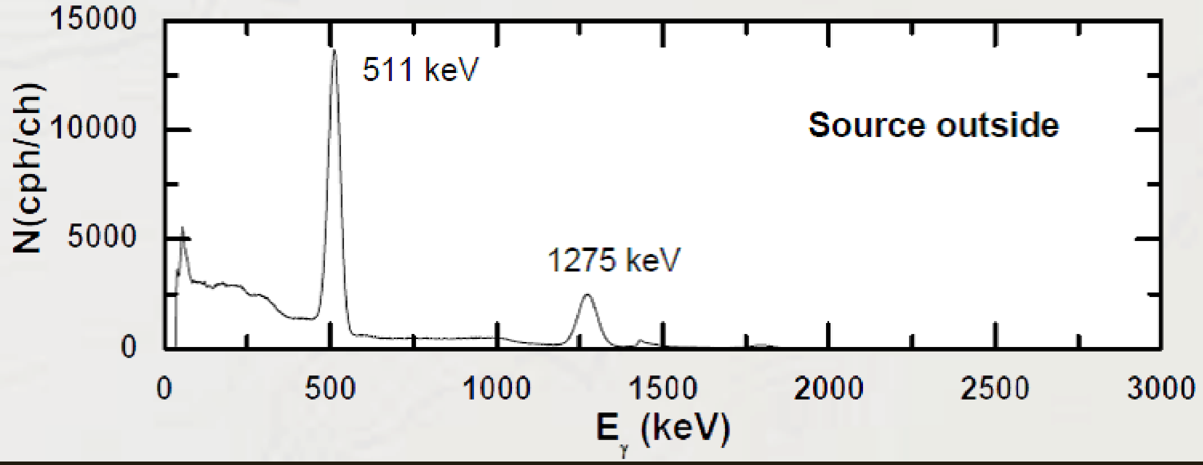

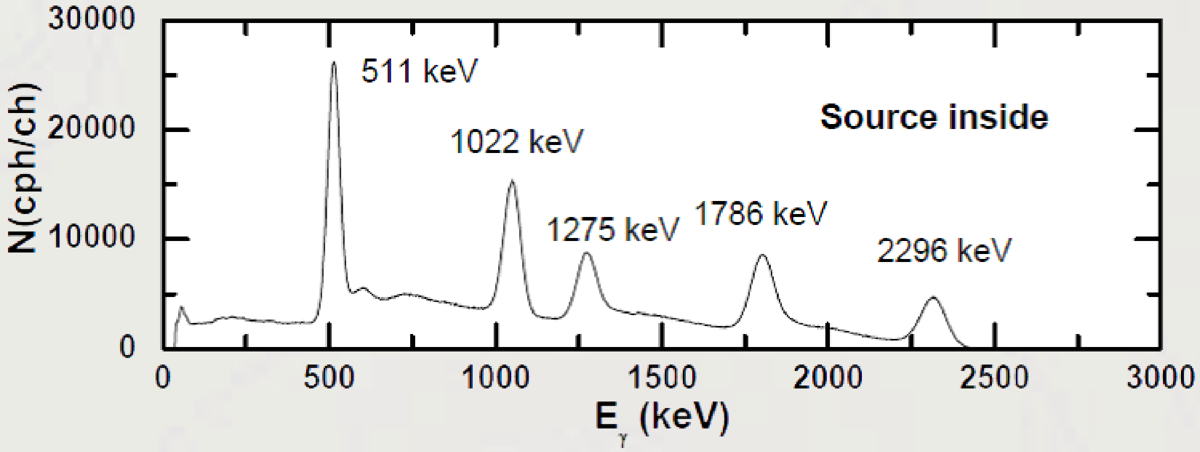

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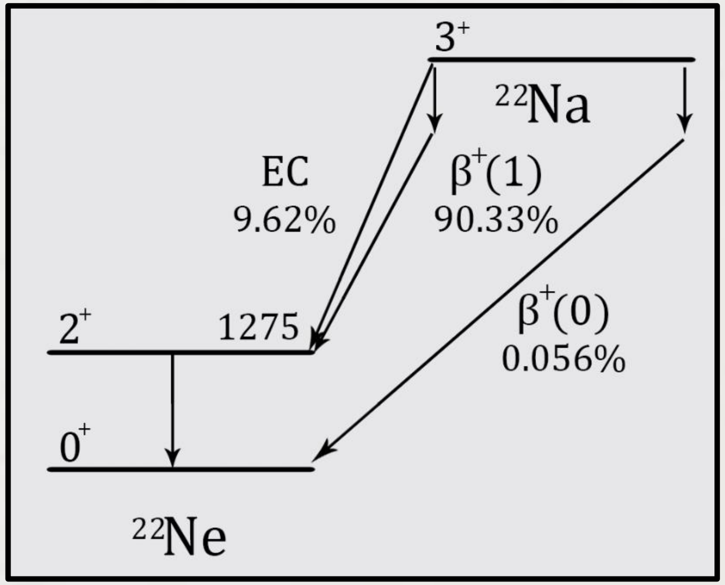
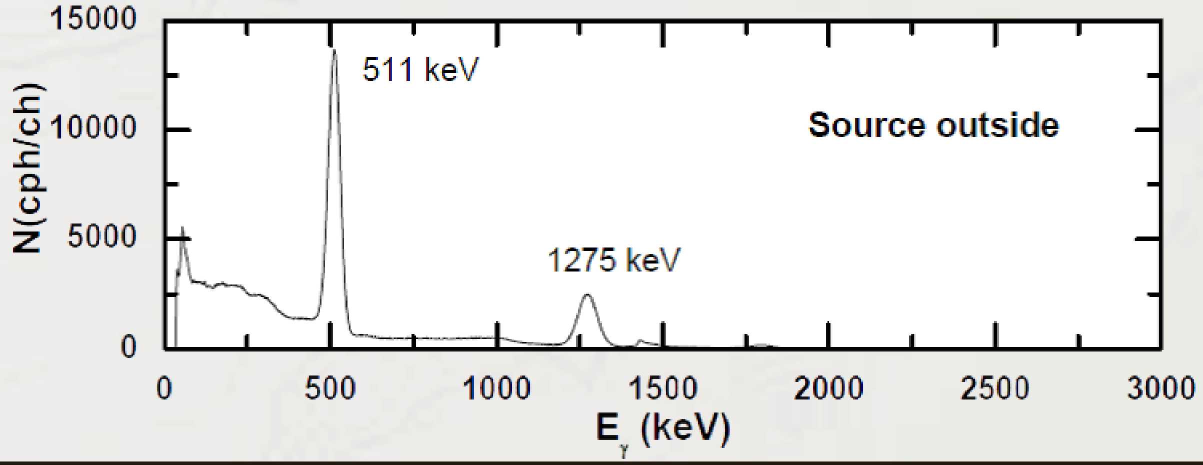


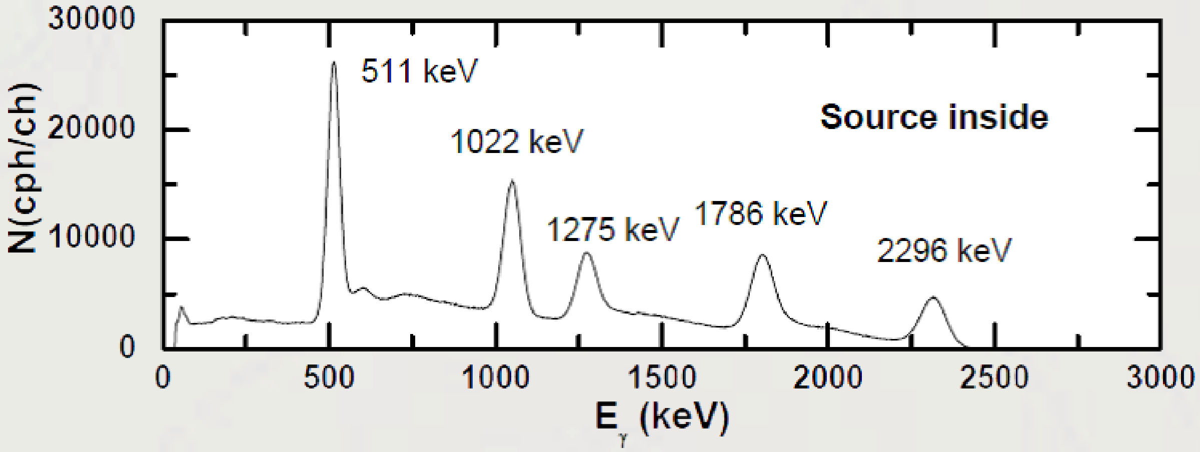
Effects of well counter (almost 4π geometry)

1022 keV= 2*511 keV

1786 keV = 511 keV + 1275 keV

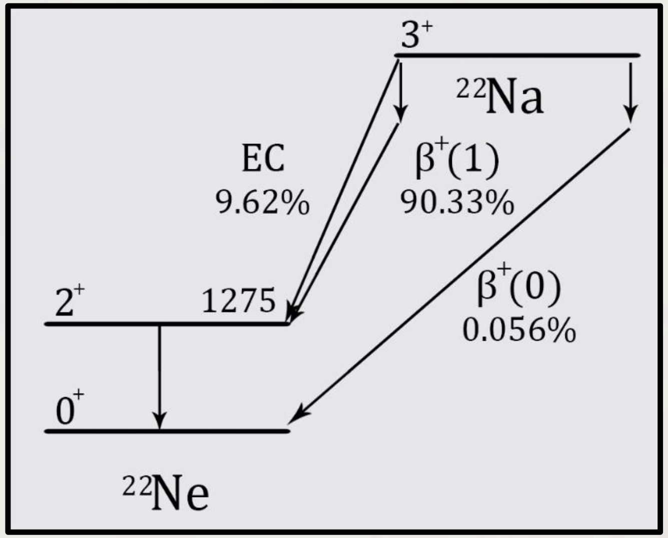
2296 keV = 2*511 keV +1275 keV





Monte Carlo simulations

Three independent simulations:

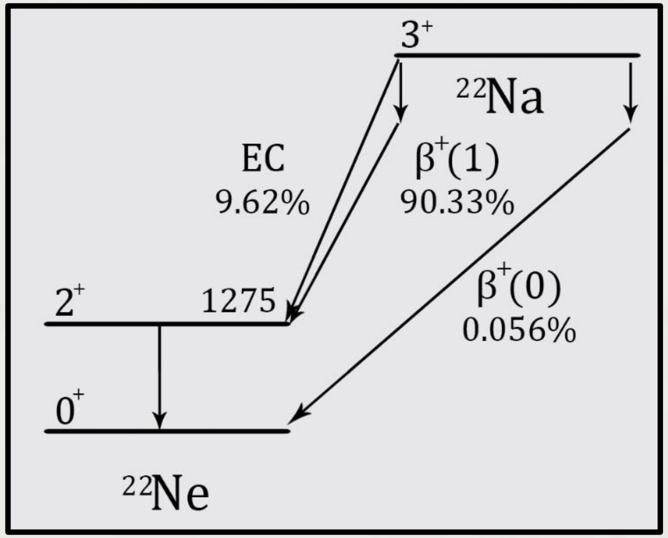


Monte Carlo simulations

Three independent simulations:

2x β$^+$ emission by ^{22}Na: $P^0_{e^+}$ and $P^1_{e^+}$

1275 keV ^{22}Ne de-excitation: P_γ



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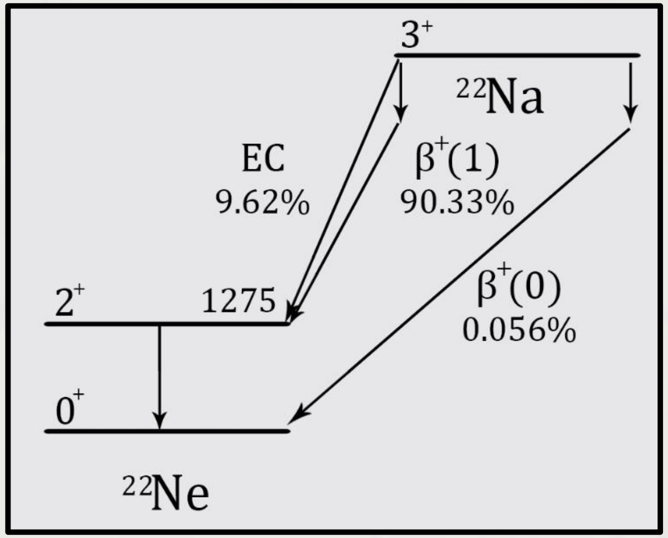


Three independent simulations:

2x β^+ emission by ^{22}Na: $P_{e^+}^{0}$ and $P_{e^+}^{1}$

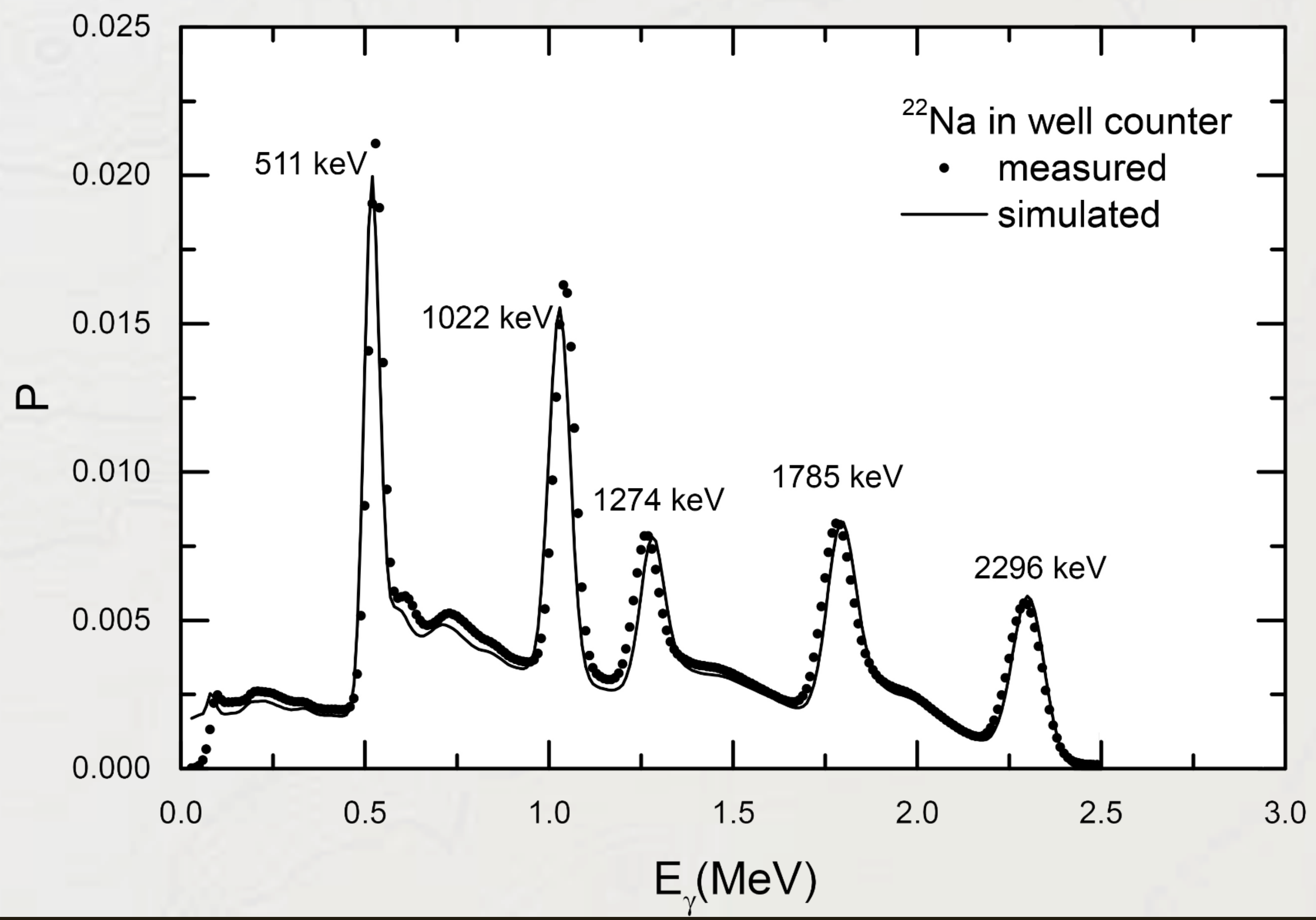
1275 keV ^{22}Ne de-excitation: P_{γ}

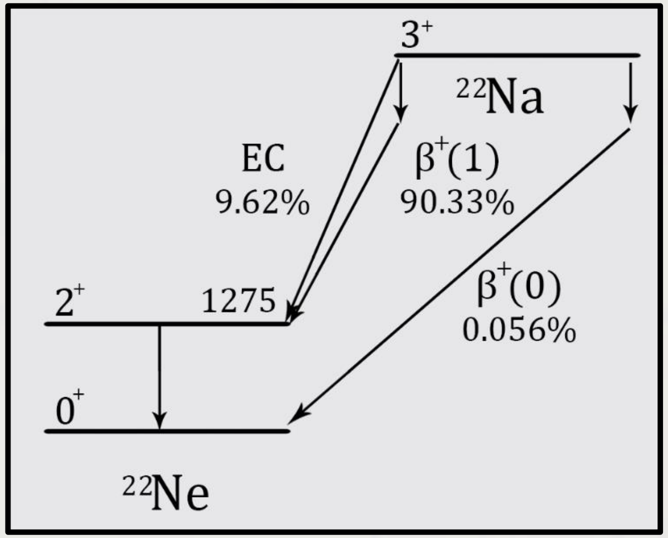
Output spectra consist of true coincidences:

$$P(i) = BR_{\beta^+(1)}\left[P_{e^+}^{1}(i)\left(1-\Sigma_{\gamma}\right) + P_{\gamma}(i)(1-\Sigma_{e^+}) + P_{CS}(i)\right] + BR_{\beta^+(0)}P_{e^+}^{0}(i) + BR_{EC}P_{\gamma}(i)$$

$$P_{CS}(i) = \sum_{j=0}^{i} P_{\gamma}(j)\, P_{e^+}^{1}(i-j)$$

Monte Carlo simulations





Double and triple coincidences

Contributions by:

- Hard- and software (PMT, MCA)
- Detector (Crystal)
- Source (Spectrum)

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- Triples: R(l) $= \sum_{k=0}^{k<l} Q(k)S(l-k)$ 1≤l≤6144

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- It should be realized that decay of doubles and triples: 2λ, 3λ

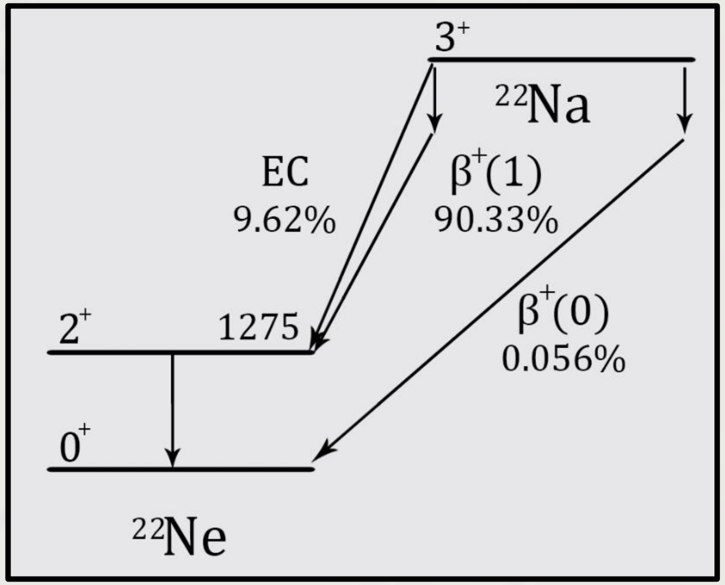
Double and triple coincidences

IMPORTANT

Singles are **TRUE** coincidences

Doubles and Triples are **ACCIDENTAL** coincidences

- Singles : S(j); $\sum_j S(j) = 1$
- Doubles: Q(k) $= \sum_{j=0}^{j<k} S(j)S(k-j)$
- Triples: R(l) $= \sum_{k=0}^{k<l} Q(k)S(l-k)$



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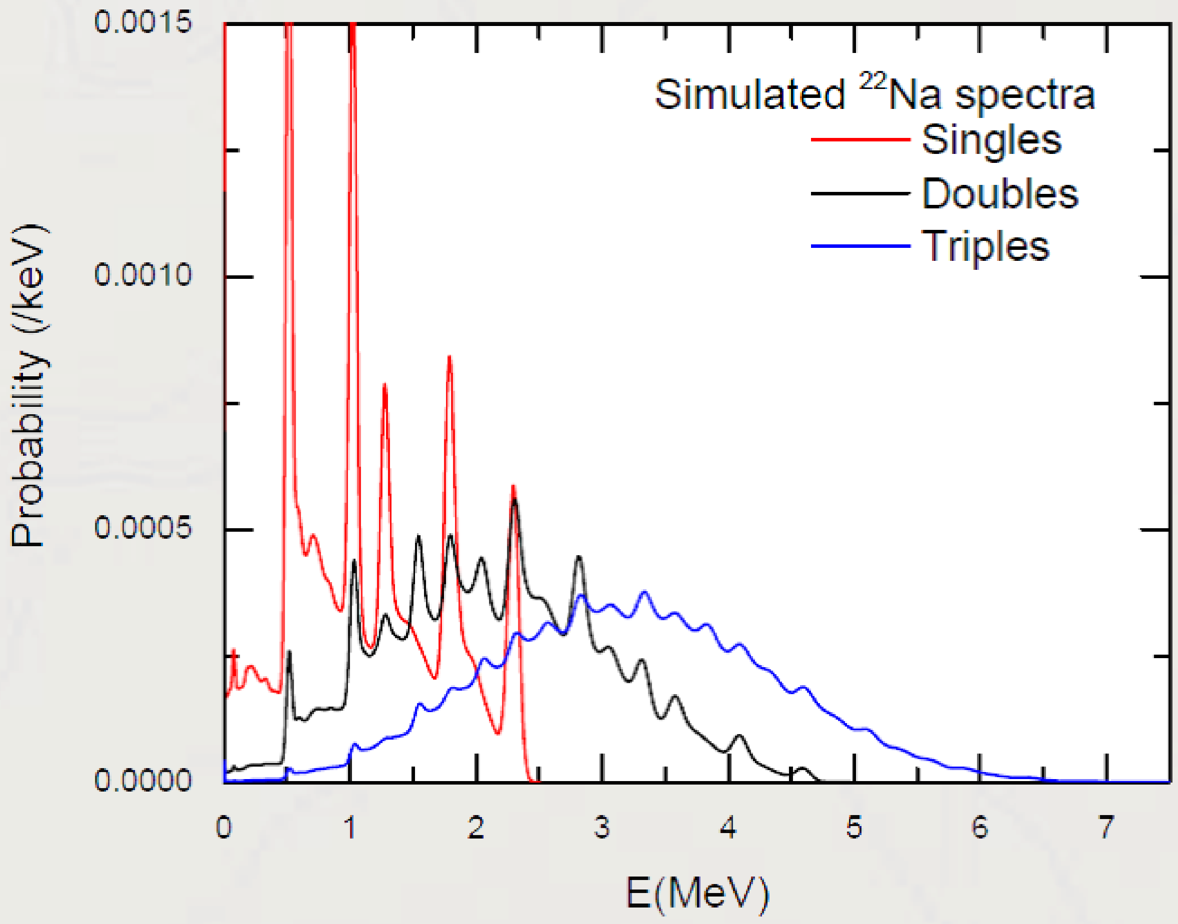
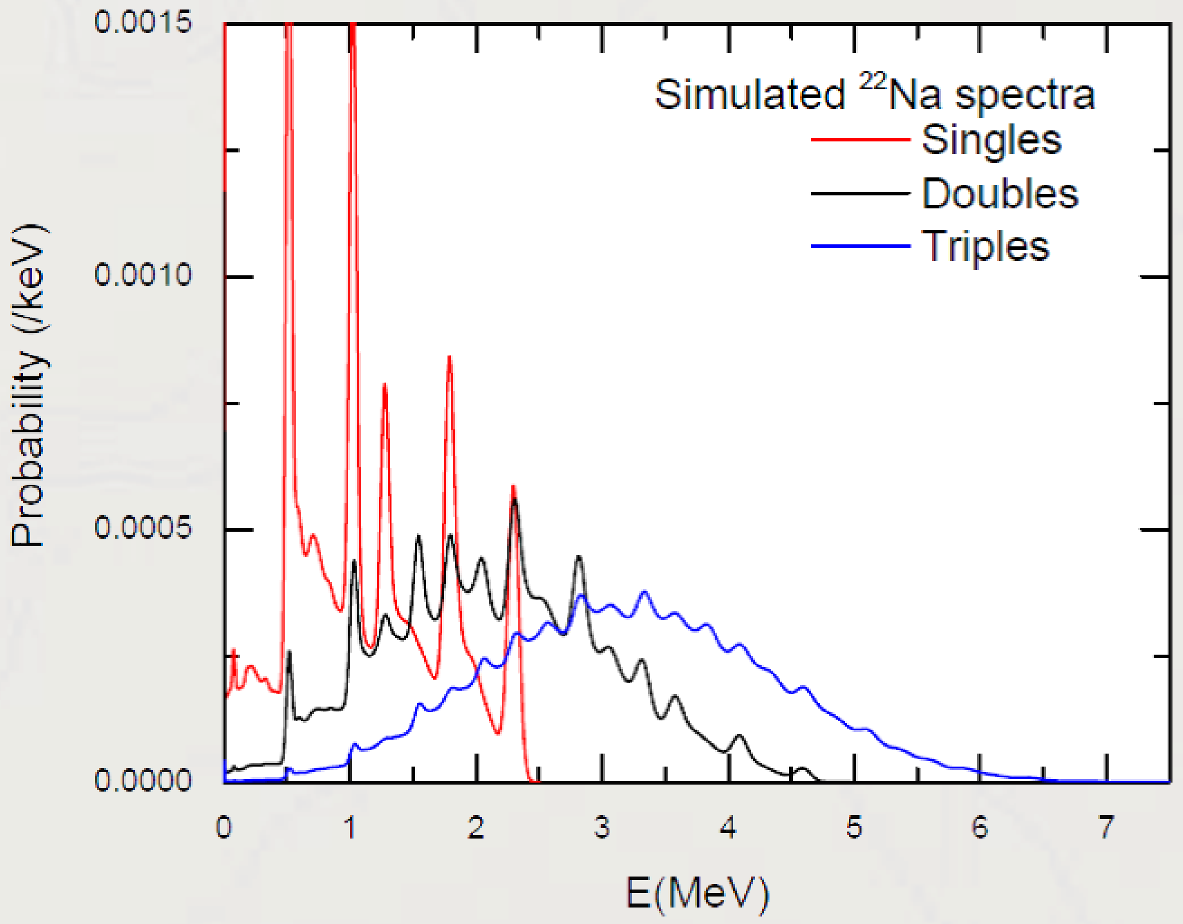
Single, double and triple spectrum



(Spectra are normalized to unity)

Single, double and triple spectrum

- Resulting spectra contain peaks



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Single, double and triple spectrum

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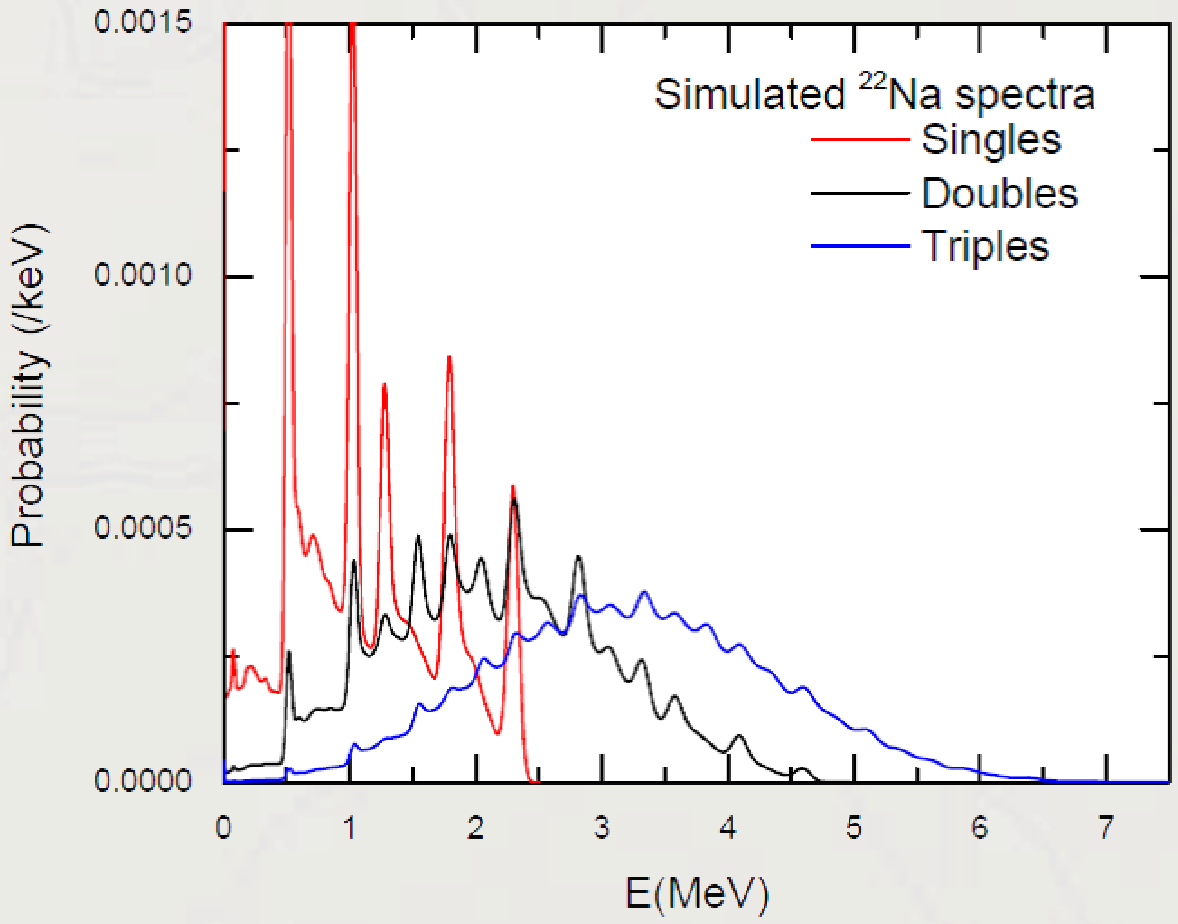
- Peaks of singles match with the doubles and triples



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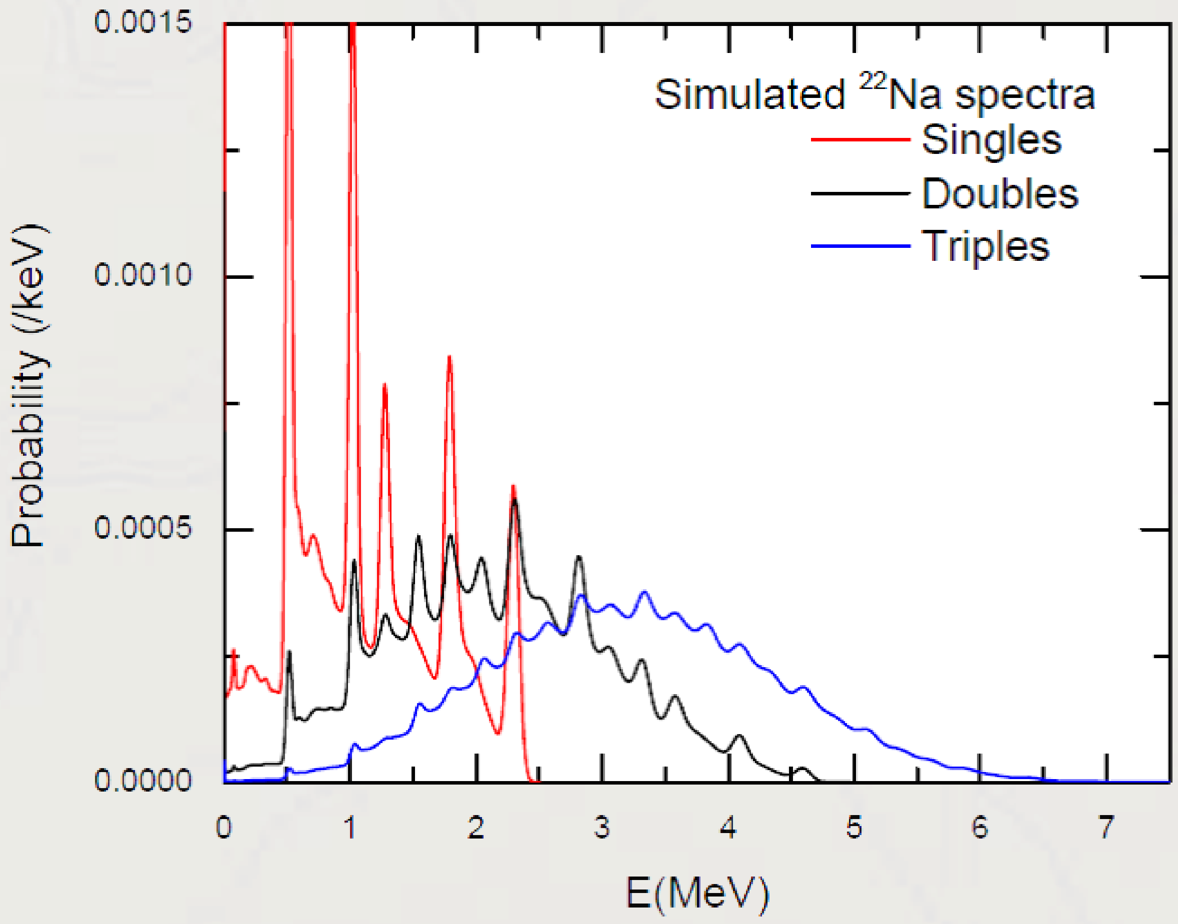
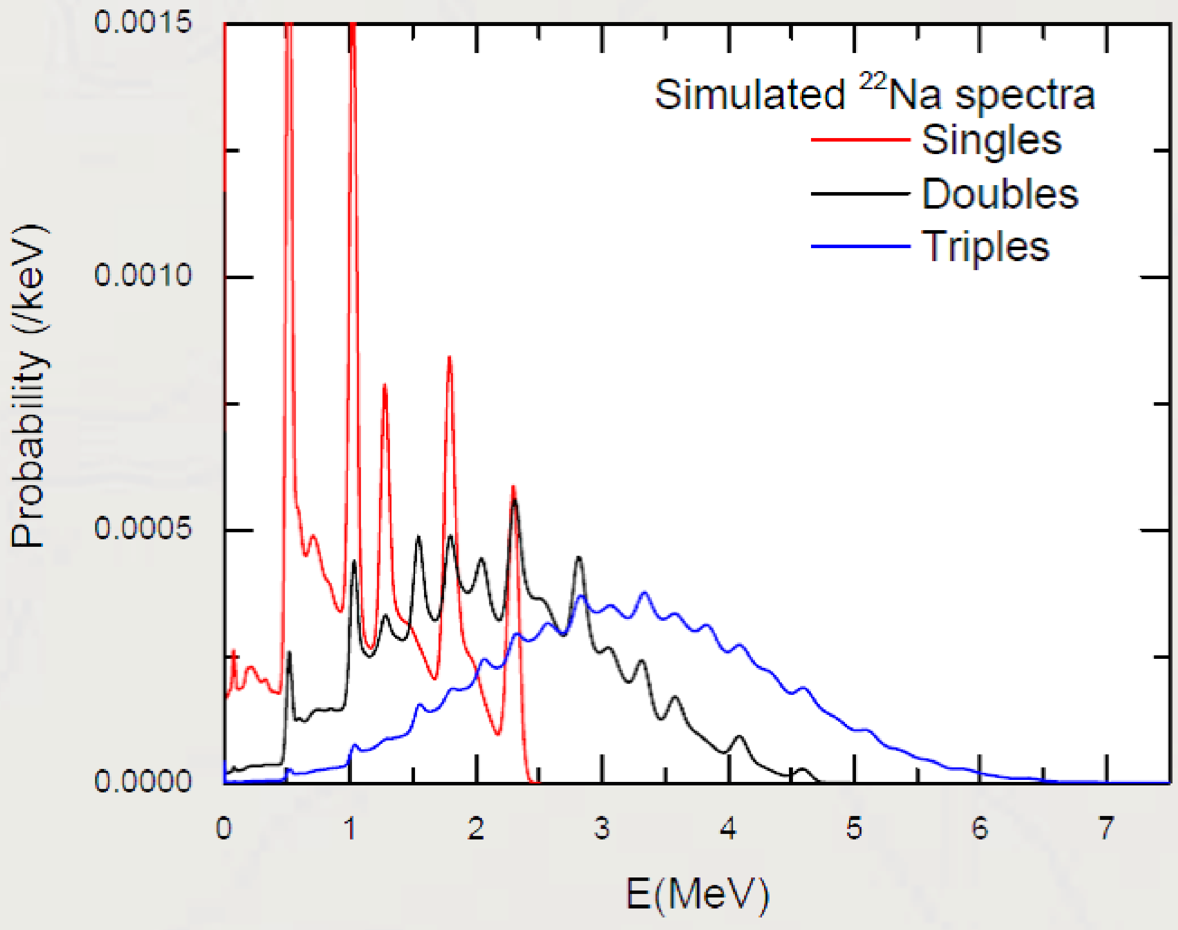


(Spectra are normalized to unity)

Single, double and triple spectrum

- Resulting spectra contain peaks
- Peaks of singles match with the doubles and triples
- New peaks occur
- Above 2.5 MeV: only coincidences



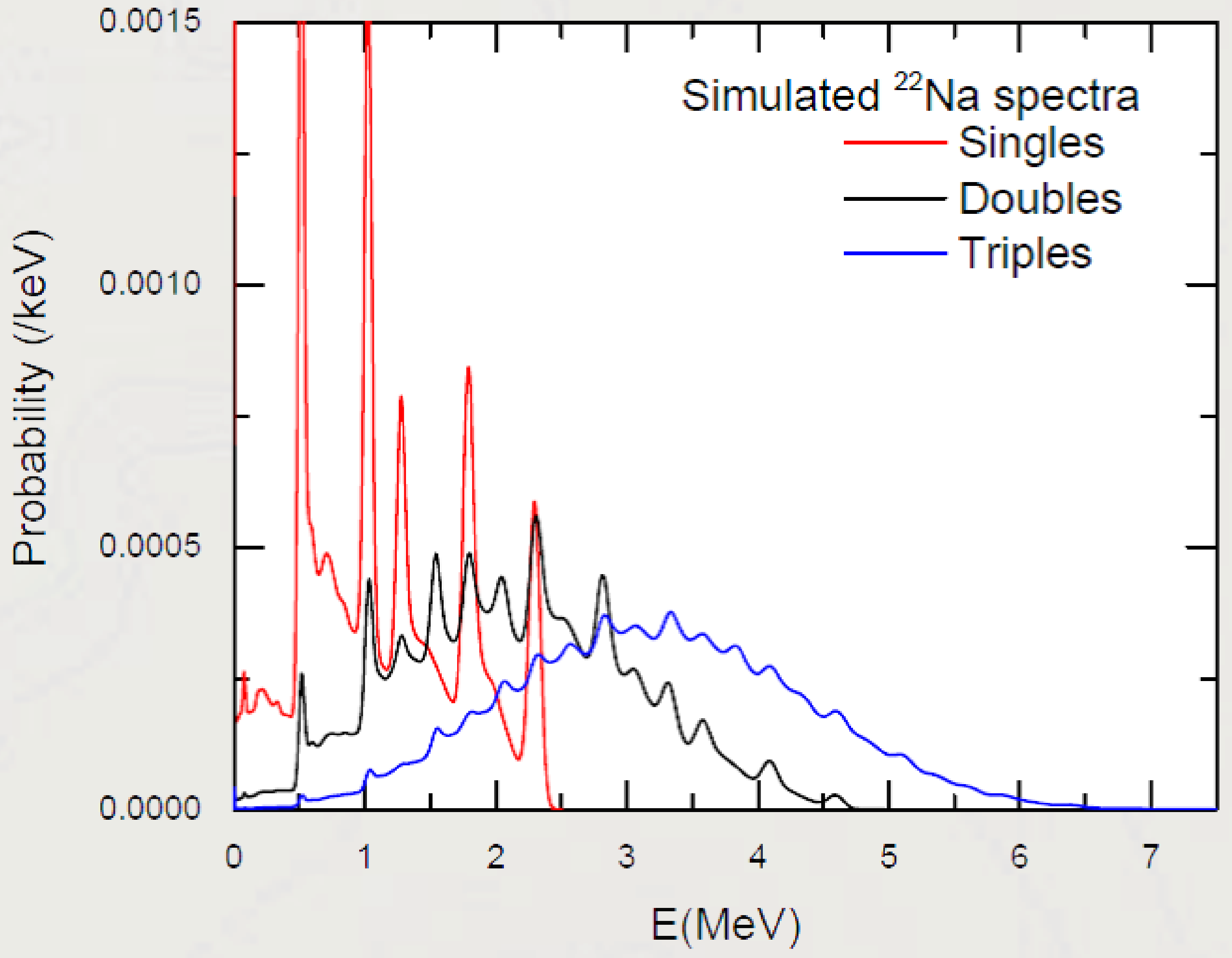
(Spectra are normalized to unity)

Validation

- **Koeberg measurements**

1. Subtract Background

(2. Correct for decay)



Validation

- **Koeberg measurements**

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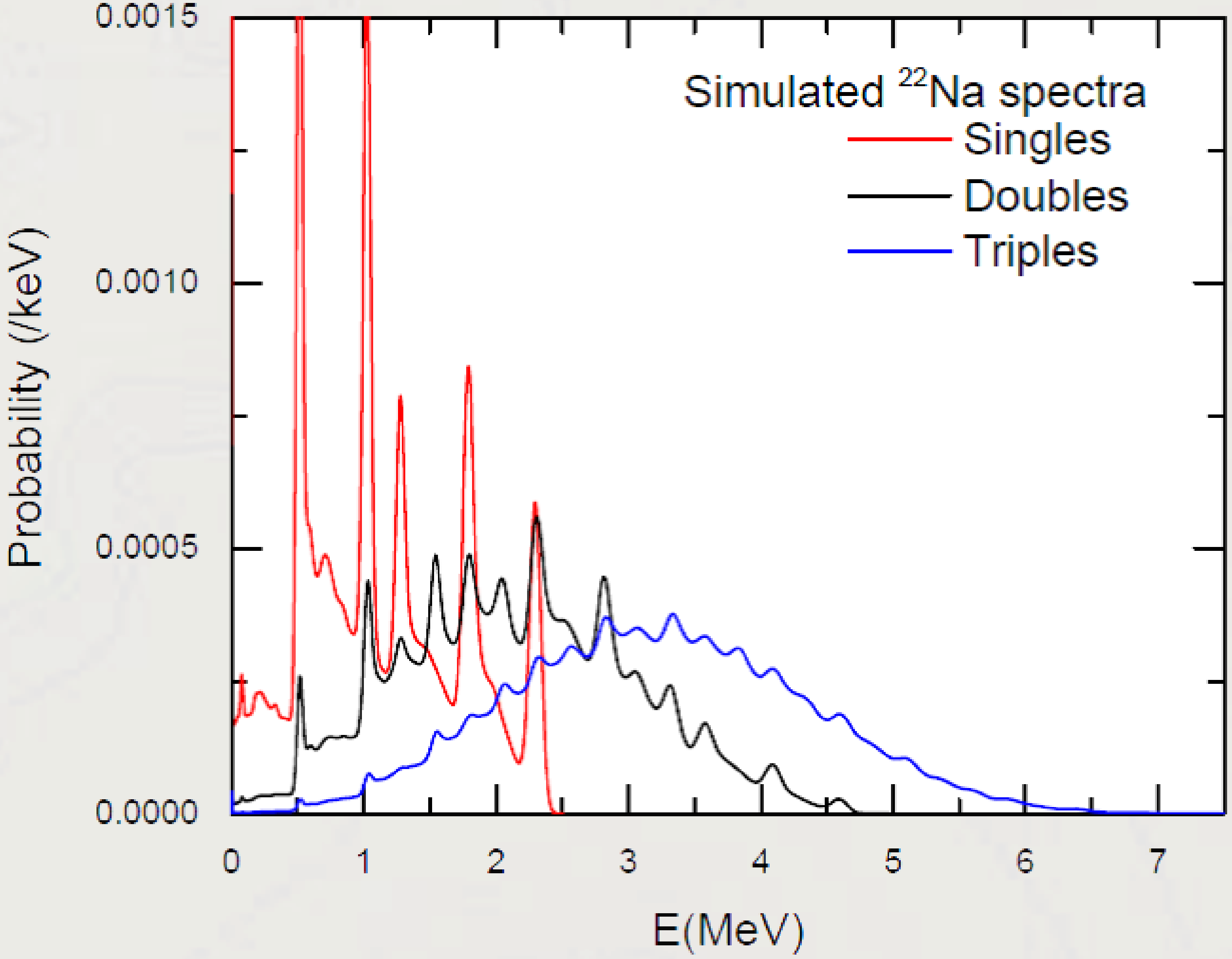
- RoI analysis: 2708 – 2998 keV



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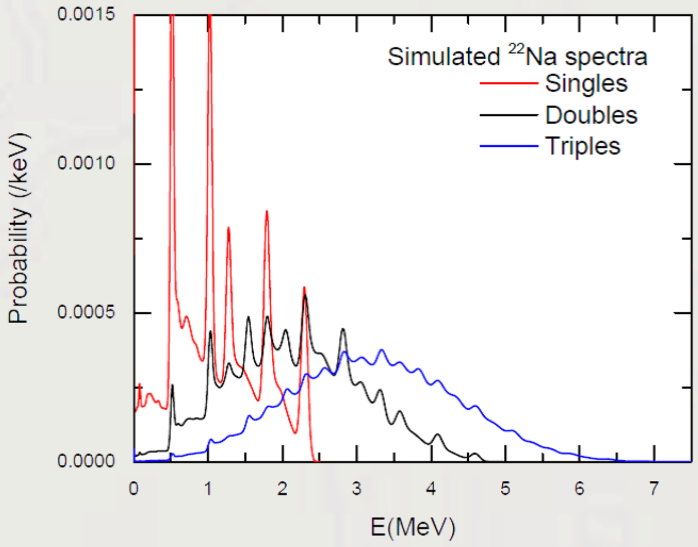
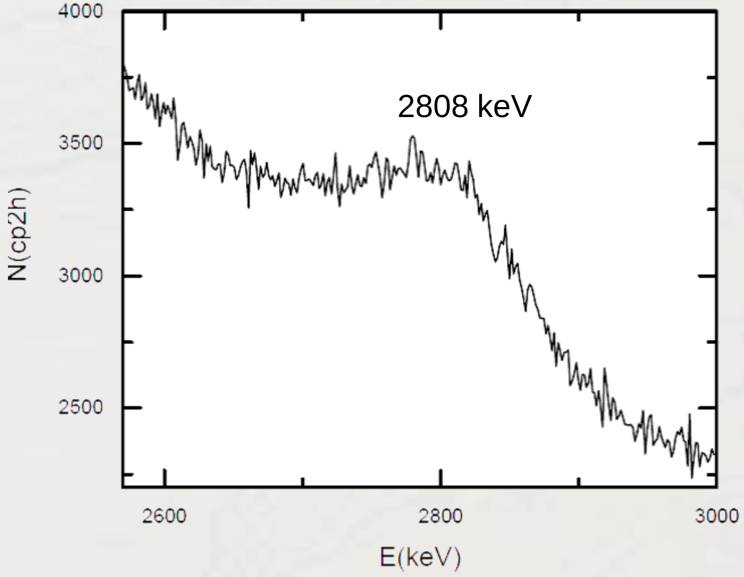


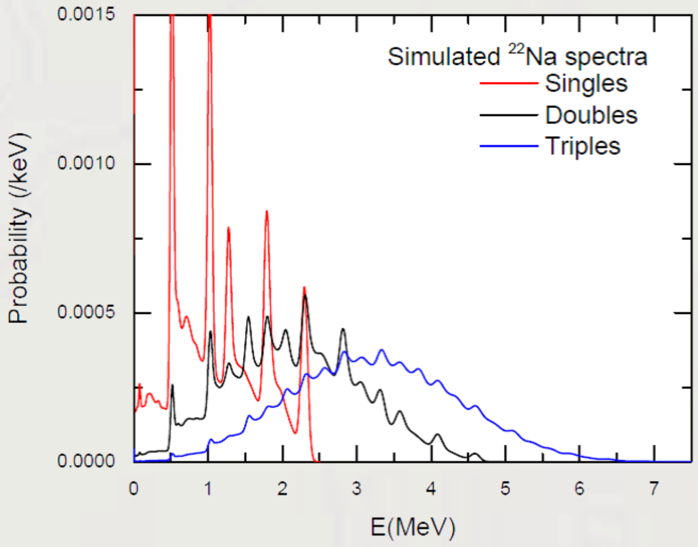
Simulated ^{22}Na spectra
Singles
Doubles
Triples
Probability (/keV)
0.0015
0.0010
0.0005
0.0000
0
1
2
3
4
5
6
7
E(MeV)

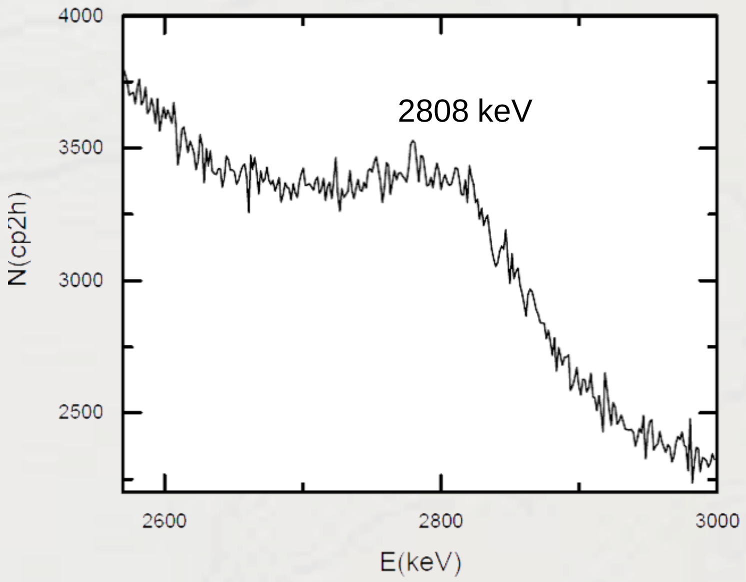


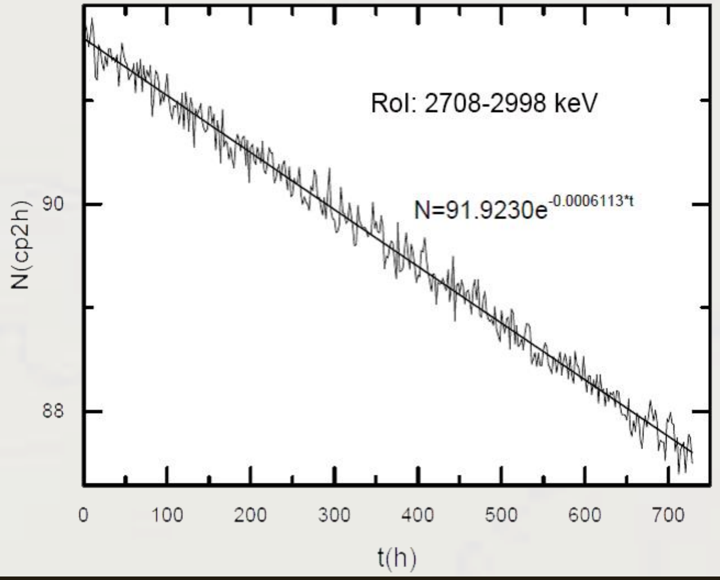
2808 keV
N(cp2h)
4000
3500
3000
2500
2600
2800
3000
E(keV)

Validation

- $t_{1/2}$ for ^{22}Na= 2.6027 Year ($\lambda = 3.0381*10^{-5}\ h^{-1}$)

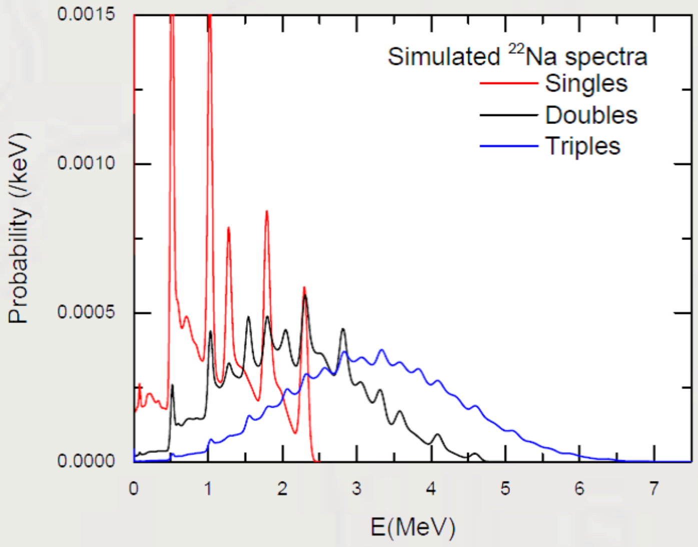


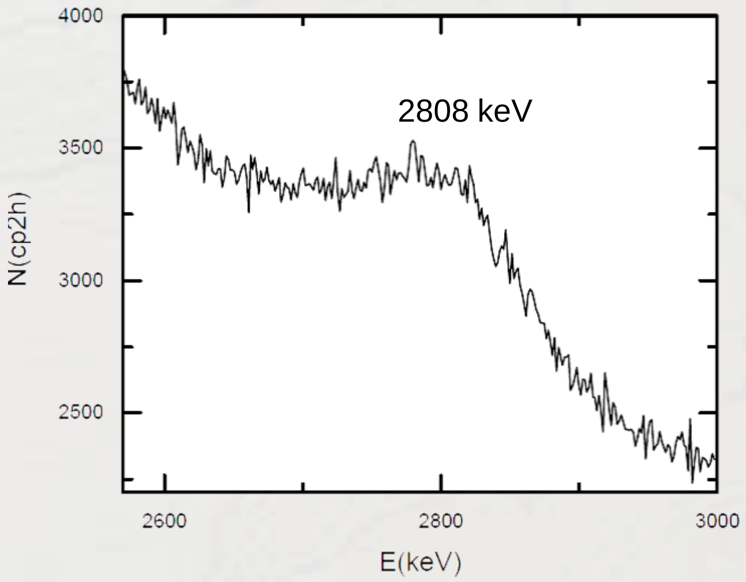


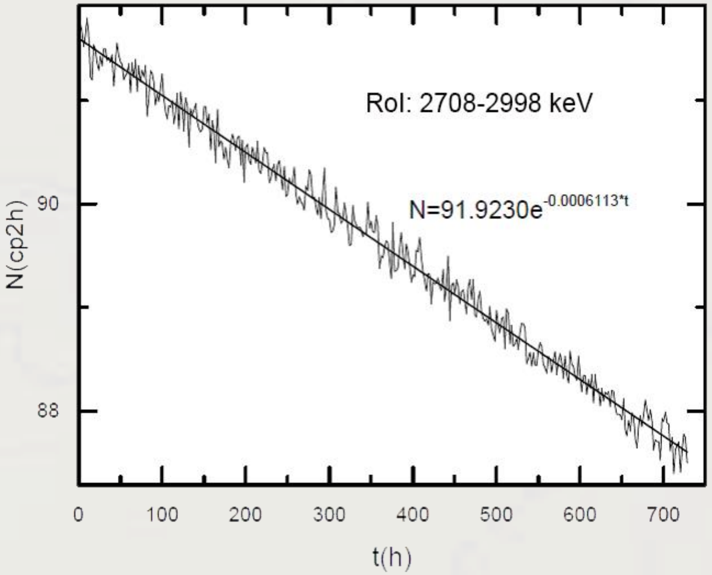


Validation

- $t_{1/2}$ for ^{22}Na= 2.6027 Year (λ = 3.0381*10^{-5} h^{-1})
- Fit gives as result: λ = 6.1130*10^{-5} h^{-1}
- Indicating doubles and triples







Validation

- Contribution of doubles and triples:

$$N(t) = B_0 N_{norm} e^{-2\lambda t}\left[N_Q + B_0 e^{-\lambda t} N_R\right]$$

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B_0 = Initial rate doubles (at t=0)
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Koeberg results:

2.2% contribution of multiples
and from that percentage 2.3% contribution of triples

Validation

- Contribution of doubles and triple

$$N(t) = B_0 N_{norm} e^{-2\lambda t}\left[N_Q + B_0 e^{-\lambda t} N_R\right]$$

Interesting

50 kBq = 1 count/20µs

→ 2.2%*20µs = resolving time of 0.5 µs

MCA = 10 MHz = 0.1 µs

Thus we need 5 samples to determine a count.

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Impact on spectral shape

- Analysis of whole spectrum (RoI 150-2520 keV):

Impact on spectral shape

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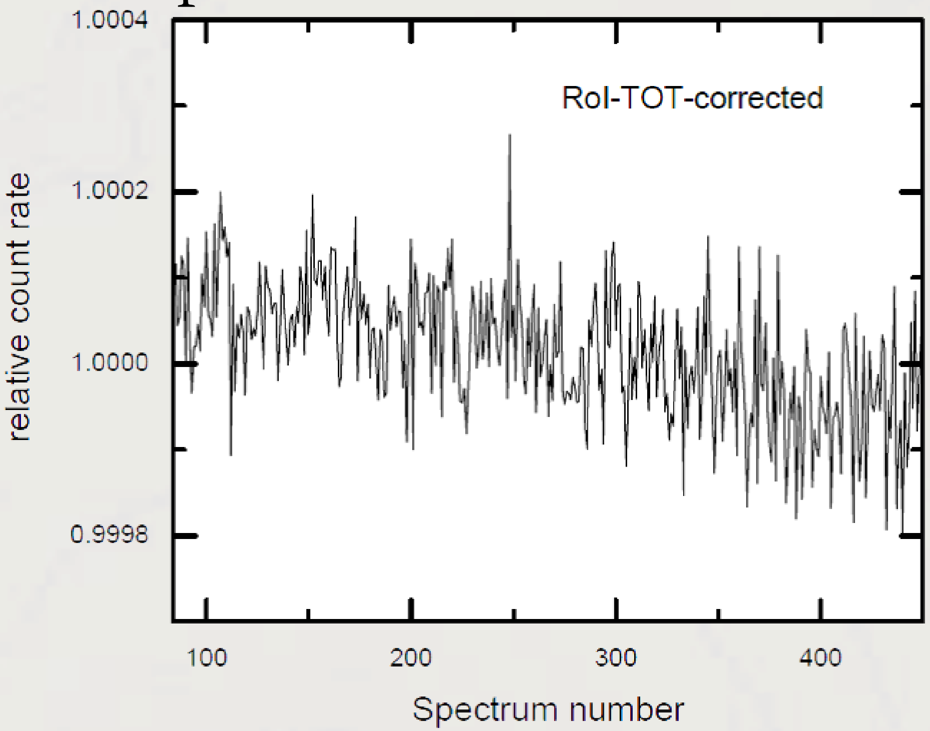

Result (λ for ^{22}Na = $3.0381 \cdot 10^{-5}$ h^{-1})

Correction	λ from fit	Error
None	$3.0996 \cdot 10^{-5}$	-2.02%
Doubles and triples	$3.0579 \cdot 10^{-5}$	-0.65%

Conclusion

- MC simulations are able to produce coincidental spectra
  - True coincidences and higher order coincidences
- Correction necessary for experiments such as the Koeberg experiments
- Correction not sufficient for Koeberg experiments
- Higher order coincidences are a measure for MCA performance

Outlook:

- Incorporate this in FSA
- Extend this to other nuclides

Thank you for your attention

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